Find a function $y = f(x)$ with the following properties:

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(i) $$\int \frac{d^2y}{dx^2}\,dx = \frac{dy}{dx} + C_1$$

$$\int \left(\frac{dy}{dx} + C_1\right)dx = y + C_1x + C_2$$

$$\int 6x\,dx = 6\frac{x^2}{2} + C_1$$

$$\int (3x^2 + C_1)\,dx = 3\frac{x^3}{3} + C_1x + C_2$$

$$= x^3 + C_1x + C_2$$

$$\Rightarrow y = x^3 + a_1x + a_2$$

(ii) $x=1,\ y=1$

$$\Rightarrow 1 = 1^3 + a_1(1) + a_2$$

$$a_1 + a_2 = 0$$

$$a_2 = -a_1$$

$$\Rightarrow y = x^3 + ax - a$$

$\frac{dy}{dx} = 0$ at $(1,1)$

$$\Rightarrow 3(1)^2 + a = 0$$

$$a = -3$$

$$\therefore y = x^3 - 3x + 3$$